

Measuring the distribution of current fluctuations through a Josephson junction with very short current pulses

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Abstract. – We propose to probe the distribution of current fluctuations by means of the escape probability histogram of a Josephson junction (JJ), obtained using very short bias current pulses in the adiabatic regime, where the low-frequency component of the current fluctuations plays a crucial role. We analyze the effect of the third cumulant on the histogram in the small skewness limit, and address two concrete examples assuming realistic parameters for the JJ. In the first one we study the effects due to fluctuations produced by a tunnel junction, finding that the signature of higher cumulants can be detected by taking the derivative of the escape probability with respect to current. In such a realistic situation, though, the determination of the whole distribution of current fluctuations requires an amplification of the cumulants. As a second example we consider magnetic flux fluctuations acting on a SQUID produced by a random telegraph source of noise.

Introduction. – The electronic transport properties of a mesoscopic conductor are completely characterized by its Full Counting Statistics (FCS), introduced in Ref. [1] and defined as the probability distribution for the transfer of charges over a certain time interval. At zero temperature and in the absence of interaction, for example, quantum transport is characterized by a binomial distribution. This contrasts with the Poissonian statistics, characteristic of classical uncorrelated particles, which is recovered in the tunneling limit [2]. The experimental determination of FCS, unfortunately, is a difficult task. Motivated by the expertise developed for current-noise measurements, one possibility is to adopt the strategy of building up the FCS through the measurement of the various cumulants of the distribution, noise being the second one. The third cumulant was indeed directly measured in Refs. [3, 4]. The alternative possibility is the direct determination of the entire FCS [5]. Recently, various proposals have been put forward in this direction making use of the high sensitivity to fluctuations of current-biased Josephson junctions (JJ) [8–17]. In this Letter we are interested in the second strategy and we analyze the case in which the measurement is performed applying very short current pulses, which allows to access a regime never explored before.

According to the RCSJ model [18], the dynamics of the

phase difference across a current-biased JJ is equivalent to that of a particle in a washboard potential, the phase playing the role of the spatial coordinate. In the harmonic approximation, the wells of such a tilted sine potential are characterized by the plasma frequency ω_p . The wells are separated by barriers if the current applied to the JJ is smaller than the superconducting critical current I_c . In such a case, the phase-particle gets trapped in a well of the potential, causing the voltage across the JJ to vanish (supercurrent state). Escape from the well, which will cause the development of a finite voltage across the JJ (transition to the resistive state), can occur through two mechanisms. At low temperatures ($T \ll \hbar\omega_p/k_B$), the only possibility is macroscopic quantum tunneling (MQT) through the barrier which separates two successive wells. The effect of thermal fluctuations on the escape rate of the MQT was considered by Martinis and Grabert [19], who found an exponential enhancement of the rate proportional to T^2 for ohmic damping. The second mechanism, thermal activation (TA), is due to excitations of the particle that will allow the latter to overcome the barrier top. In the absence of perturbations, these are produced by thermal fluctuations at large temperatures ($T > \hbar\omega_p/k_B$).

In fact, the probability of escape from a potential well is sensitive to perturbations affecting the system, such

as bias-current fluctuations. This is precisely the phenomenon current fluctuations detection is based on. Indeed, by adding to the bias current, assumed to be constant, the fluctuating component of the current under investigation, it has been shown that the monitoring of the voltage appearing across the JJ can be used to characterize such fluctuations, either in the TA or MQT regime. Regarding the former, in Ref. [8] an array of overdamped JJ which realizes a threshold detector was proposed, whereas in Ref. [12] a method was presented for the determination of the third cumulant. On the other hand, MQT was exploited for the determination of the FCS in Ref. [9] and of the fourth cumulant in Ref. [11]. Successful measurements in the TA and MQT regimes were very recently reported, respectively, in Ref. [13] for the second cumulant, and in Refs. [15, 16] for the third cumulant. In this paper we are interested in hysteretic (underdamped) JJ in the MQT regime. Inspired by the experiments [20–22], we consider the situation in which the measurement is performed by applying a sequence of very short current pulses (of duration Δt of the order of a few nanoseconds) [23]. This, together with a proper filtering of the frequency components of the current fluctuations, allows to realize a separation of time scales, as we shall argue in the following (see Fig. 1(a)).

Current fluctuations whose frequency ω is larger than the plasma frequency ω_p give rise, even at zero temperature, to thermal-like activation and represent the non-adiabatic regime. For fluctuations of frequency smaller than the plasma frequency, escape from the well can only occur through MQT, which turns out to be exponentially sensitive to current fluctuations. This is the adiabatic regime since the particle remains in the ground state of the well. Within the adiabatic regime, one can realize two different situations depending on whether one selects the current fluctuations which occur on a time scale longer (low frequency regime) or shorter (high frequency regime) than Δt . For large Δt , of the order of μs , only the high frequency (HF) regime is accessible, since long-time-scale fluctuations do not contribute appreciably (see Refs. [9, 15]). In this case the selection is obtained by placing a low-pass filter of bandwidth $\omega_b \ll \omega_p$ between the source of current fluctuations and the JJ. In this Letter we shall focus on the low-frequency (LF) regime, where Δt is of the order of a few nanoseconds [20–22] and $\omega_b \lesssim 2\pi/\Delta t \ll \omega_p$ in order for the HF components not to affect the escape probability [24]. The current will fluctuate for different pulses, remaining constant within a single pulse. We shall discuss how the distribution of current fluctuations can be extracted from the escape probability histogram and, in the small skewness limit, how the former depends on the second and third cumulants. We shall furthermore show that in some cases the influence of the current distribution in the LF regime on the probability histogram is more transparent than that relative to the HF regime. In addition we discuss in detail two different examples. We shall assume that the current fluctuations are

produced by: i) a tunnel junction; ii) a source of random telegraph noise [25].

The system. – The system we consider (see Fig. 1(b)) is a dc-SQUID, composed of a superconducting loop with two identical JJ. We consider a loop of small inductance, such that the SQUID is equivalent to a single JJ with a flux-tunable critical current. The SQUID is biased by a constant current source I and coupled to a voltage amplifier. The fluctuating component $\delta I(t)$ of the current originating from a mesoscopic conductor is fed into the SQUID through a proper filtering circuit (not shown in the figure) [26]. Such a circuit represents the environment of the SQUID which must be taken into account in the interpretation of the results of an experiment. Indeed, the environment will introduce additional contributions to the current fluctuations under investigation even exerting a back-action on the system [28, 29]. Since the exact effect is determined by the details of the circuit, in the following we shall disregard the effects of the environment and focus only on the mere effects of current fluctuations.

Current fluctuations are characterized by the distribution $\rho(\delta I)$ which can be written as a Fourier transform $\rho(\delta I) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk e^{-ik\delta I} \phi_{\delta I}(k)$ of the characteristic moment-generating function $\phi_{\delta I}(k) = \exp(\sum_{n=2}^{\infty} \frac{(ik)^n}{n!} c_n)$, where c_n are the cumulants of the distribution. In particular, c_2 is related to the width ($c_2 = \langle \delta I^2 \rangle$) and c_3 to the asymmetry of the distribution ($c_3 = \langle \delta I^3 \rangle$).

The JJ, which is assumed to be underdamped, is operated as follows. A sequence of current pulses, of duration Δt and amplitude I , is applied to the JJ, and the voltage across the JJ is monitored. If, during the time interval Δt , a finite voltage across the JJ develops, the event is recorded. The ratio between the number of transitions and the total number of pulses applied is defined as the escape probability, which, as a function of the average amplitude I of the bias current, gives rise to a histogram $P(I)$ (see Fig. 1(c)). Such an escape probability histogram is exponentially sensitive to perturbations affecting the system, such as current fluctuations $\delta I(t)$ produced by the mesoscopic element or fluctuations of the magnetic flux threading the SQUID. We assume that temperature is small such that we can neglect thermal fluctuations.

For very short pulses we access the LF regime, in which the current is constant within a single pulse, but fluctuates for different pulses. Assuming ergodicity, the escape probability is given by averaging the probability in the absence of fluctuations P_0 over the distribution of current fluctuations $\rho(\delta I)$:

$$P(I) = \int_{-\infty}^{+\infty} d(\delta I) \rho(\delta I) P_0(I + \delta I), \quad (1)$$

where $P_0(I) = 1 - e^{-\Gamma \Delta t}$, $\Gamma = \Gamma(I, \Phi)$ being the escape rate in the MQT regime given by $\Gamma(I) \equiv A(I) \exp(-B(I))$, where $A(I) = 12 \sqrt{\frac{6\pi \Delta U(I)}{\hbar \omega_p(I)} \frac{\omega_p(I)}{2\pi}}$ and $B(I) = \frac{36}{5} \frac{\Delta U(I)}{\hbar \omega_p(I)}$ in the limit of low dissipation [30]. The plasma fre-

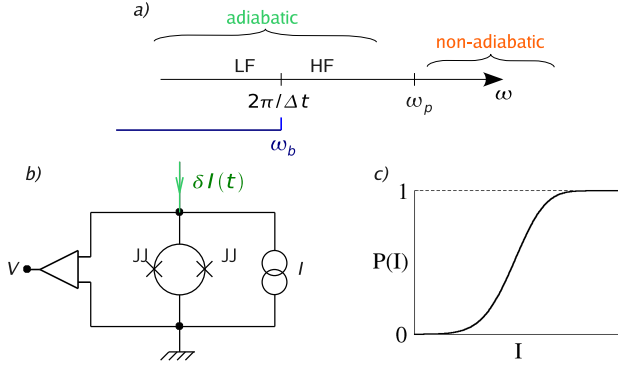


Fig. 1: a) The frequency arrow: fluctuations with frequency $\omega \ll \omega_p$ produce an adiabatic perturbation of the washboard potential, thus affecting the escape probability with exponential sensitivity. High frequency (HF) fluctuations ($\omega > 2\pi/\Delta t$), occur over the duration of a pulse giving rise to an averaged escape rate. On the contrary, for low frequency (LF) fluctuations ($\omega < 2\pi/\Delta t$), the current does not fluctuate during a pulse, but changes from pulse to pulse giving rise to an averaged escape probability. In the LF regime we assume $\omega_b \lesssim 2\pi/\Delta t \ll \omega_p$, where ω_b is the bandwidth. Non-adiabatic perturbations are due to fluctuations with frequency $\omega > \omega_p$ which give rise to thermal-like activation. b) The system consists of a current-biased dc-SQUID connected to a voltage amplifier. δI represents the fluctuating component of the current flowing through a mesoscopic element, whose current fluctuations distribution is under investigation. c) Escape probability histogram obtained by varying the bias current.

quency is given by $\hbar\omega_p = \sqrt{8E_J E_C} \left[2 \left(1 - \frac{I}{I_c} \right) \right]^{1/4}$, where $E_J = \hbar I_c / 2e$ is the Josephson energy, $E_C = e^2 / 2C_J$ is the charging energy and I_c is the critical current of the SQUID for zero magnetic flux. ΔU denotes the barrier height and it is defined as $\Delta U = \frac{2}{3} E_J \left[2 \left(1 - \frac{I}{I_c} \right) \right]^{3/2}$.

The distribution $\rho(\delta I)$ can be derived directly by deconvoluting the measured escape probability, once P_0 is known. By Fourier transforming Eq. (1) one obtains the characteristic function $\phi(k) = \tilde{P}(k) / \tilde{P}_0(k)$ where \tilde{P} (\tilde{P}_0) is the Fourier transform of P (P_0). The various cumulants of the distribution can then be calculated through the derivatives $c_n = (-i)^n \frac{\partial^n}{\partial k^n} \log \phi(k) \big|_{k=0}$. In the case where P_0 has a very sharp transition [$P_0(I) \simeq \theta(I - I_0)$] the distribution can be simply obtained by a derivation

$$\rho(x) = \frac{dP(I_0 - x)}{dx}, \quad (2)$$

so that the central moments of the distribution are obtained as $\kappa_n = \int dx x^n \rho(x)$. We wish to remark that in the LF regime the low frequency component only will contribute to the moments and hence to ρ . For example:

$$\kappa_2 = \int_0^{\frac{2\pi}{\Delta t}} \frac{d\omega}{2\pi} S_{II}(\omega) \quad (3)$$

$$\kappa_3 = \int_0^{\frac{2\pi}{\Delta t}} \frac{d\omega_1}{2\pi} \int_0^{\frac{2\pi}{\Delta t}} \frac{d\omega_2}{2\pi} S_{III}(\omega_1, \omega_2), \quad (4)$$

where

$$S_{II}(\omega) = \int_{-\infty}^{+\infty} dt e^{i\omega t} \langle \Delta I(t) \Delta I(0) \rangle \quad (5)$$

$$S_{III}(\omega_1, \omega_2) = \int_{-\infty}^{+\infty} dt_1 e^{i\omega_1 t_1} \int_{-\infty}^{+\infty} dt_2 e^{i\omega_2 t_2} \times \langle \Delta I(t_1) \Delta I(t_2) \Delta I(0) \rangle. \quad (6)$$

Higher frequency (adiabatic) components are filtered out by the external circuit.

Small skewness limit. – Neglecting the fourth and higher moments of current, and in the limit where the skewness γ of the current distribution is small ($\gamma \equiv c_3/c_2^{3/2} \ll 1$), one obtains

$$\rho(\delta I) \simeq \frac{1}{\sqrt{2\pi c_2}} \left(1 - \frac{c_3}{2c_2^2} \delta I + \frac{c_3}{6c_2^3} \delta I^3 \right) \exp\left(-\frac{\delta I^2}{2c_2}\right). \quad (7)$$

We shall refer to the escape current I_{esc} as the current relative to $P(I_{\text{esc}}) = 1/2$, and to the width of the transition as $\Delta I = I_{0.9} - I_{0.1}$, with I_x defined by $P(I_x) = x$.

For the sake of definiteness, we shall assume that P_0 has a very sharp transition (with respect to the values of c_2/I_0 considered), so that we approximate the probability with a theta function $P_0(I) = \theta(I - I_0)$. Let us now discuss the behavior of the probability histogram in the LF regime. For $c_3 = 0$ (Gaussian noise), the escape current does not depend on c_2 , while the width of the transition ΔI increases linearly with $\sqrt{c_2}$. More precisely, $\Delta I = 2\sqrt{2c_2} \text{Erf}^{-1}(4/5)$, where Erf^{-1} indicates the inverse of the error function. Now, a finite skewness is expected to produce an increase (decrease) of the escape current for $\gamma > 0$ ($\gamma < 0$). We calculate the escape probability and determine I_{esc} and ΔI as a function of γ for different values of c_2 . As shown in Fig. 2, I_{esc} increases linearly with γ , the slope being proportional to $\sqrt{c_2}/I_0$ in such a way that $(I_{\text{esc}} - I_0)/\sqrt{c_2} = \text{const} \cdot \gamma$. Since ΔI at $\gamma = 0$ depends on the value of c_2 , in Fig. 3 we report the plot of the relative width change $\Delta I_r = \Delta I - \Delta I(\gamma = 0)$ as a function of γ . A quadratic dependence on γ is found, with a coefficient proportional to $\sqrt{c_2}/I_0$ so that $\Delta I_r/\sqrt{c_2} = \text{const} \cdot \gamma^2$. Note however that the dependence is very weak: 0.01% escape current change and 0.1% transition width change in the range of γ considered. We wish to mention that the results in the case of a finite-width transition, in the absence of fluctuations, are qualitatively equal. For example, in the experimentally relevant case in which $\Delta I_{\text{int}}/I_0 = 5 \times 10^{-3}$, where ΔI_{int} is the intrinsic width of P_0 , we find the same dependence of I_{esc} and ΔI on γ . The only quantitative difference is that the values of ΔI are shifted upwards, while the curve of I_{esc} is slightly shifted to lower values.

It is interesting to compare the above results with the ones relative to the HF regime, in which the current fluctuates within a single pulse. In such a case the escape probability is given by averaging the escape rate over the noise distribution $\rho(\delta I)$ of Eq. (7):

$$P(I) = 1 - e^{-\langle \Gamma(I) \rangle \Delta t} = 1 - e^{-\int_{-\infty}^{+\infty} d(\delta I) \rho(\delta I) \Gamma(I + \delta I) \Delta t}. \quad (8)$$

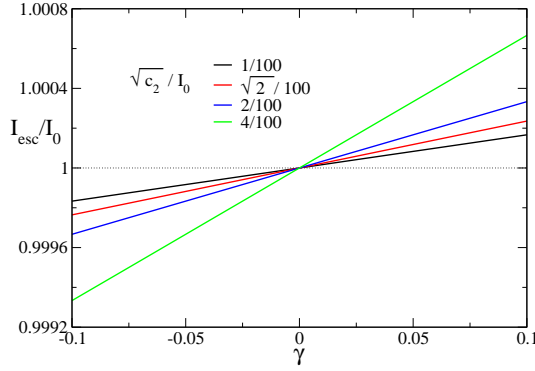


Fig. 2: I_{esc}/I_0 as a function of γ for several values of c_2 . A linear dependence is found with slope proportional to $\sqrt{c_2}/I_0$.

It turns out that the width of the transition ΔI does not depend on γ so that the skewness affects the escape current only. However, since the influence of c_2 on the escape current is strong, it is difficult to isolate the shift in the escape current due to a change of γ .

In Ref. [27] it was shown that, in the HF regime, the effect of the third moment on the escape rate can be singled out by inverting the polarity of the current I . Such inversion results in a mere shift of the escape current which allows, in principle, to evaluate the value of γ . We apply a similar analysis to the LF regime finding, interestingly, that it is the escape probability as a whole that is affected by the presence of the third moment. For the average escape probability we obtain

$$P^\pm(I) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx P_0(I + \sqrt{c_2}x) \exp(-x^2/2) \mp \frac{\gamma}{2} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx P_0(I + \sqrt{c_2}x) x(1 - x^2/3) \exp(-x^2/2), \quad (9)$$

where the subscript \pm refers to positive (negative) current polarities. The values of γ and c_2 can be determined if P_0 is known. In particular, when P_0 can be approximated by a theta function, the value of γ can be determined from

$$P^+(I) - P^-(I) = -\gamma \frac{e^{-\frac{(I-I_0)^2}{2c_2}}}{2c_2\sqrt{2\pi}} (I - I_0)^2, \quad (10)$$

while c_2 can be determined from the sum $P^+ + P^-$. It turns out that the effect of the inversion of the current polarity is smaller in the LF regime than in the HF regime. However, while for LF γ can be evaluated directly by comparing two histograms (see Eq. (10)), for HF γ can be determined only indirectly from the escape rates and the histogram derivative [27]. Finally we note that spurious current asymmetries may arise in the SQUID as a result of parasitic magnetic fields. To avoid this phenomenon one should replace the SQUID with a JJ as detector.

Tunnel junction. — Let us consider a concrete example in which the mesoscopic element is a tunnel junction.

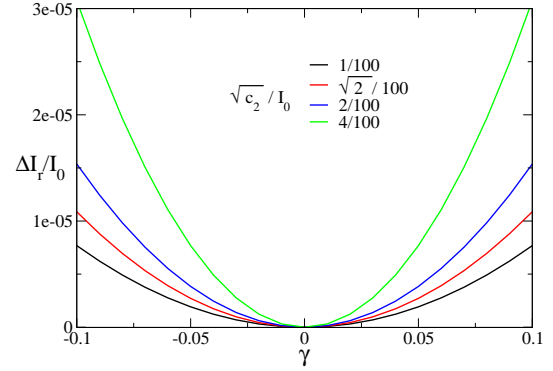


Fig. 3: The normalized relative width change $\Delta I_r/I_0$ is plotted as a function of γ for several values of c_2 .

The current flowing through it is distributed according to

$$\rho_{\text{tun}}(I) = \frac{T}{e} \frac{\bar{N}^{\frac{T}{e}} I e^{-\bar{N}}}{\Gamma(\frac{T}{e} I + 1)}, \quad (11)$$

where $\bar{N} = T/e\bar{I}$ is the mean number of electron transferred in a time T , \bar{I} is the mean current, e the electronic charge, and Γ is the Euler Gamma function. Despite the fact that the FCS of a tunnel junction is Poissonian, with all cumulants equal to \bar{N} , the cumulants of the current distribution ρ_{tun} are given by $c_n = \bar{I}(e/T)^{n-1}$, *i.e.* decreasing with increasing n . It is interesting that for \bar{N} as large as 100, ρ_{tun} is easily distinguishable from the Gaussian distribution of equal c_2 . Such small values of \bar{N} can be achieved, in our detection scheme, since the time T can be identified with $2\pi/\omega_b$, which is taken to be of the order of the pulse duration Δt . What limits the smallness of \bar{N} is, firstly, the minimum achievable value Δt , and secondarily the maximum value of ω_b , which must be smaller than ω_p . As an example, in Fig. 4, we plot the function $\bar{\rho}_{\text{tun}}(\delta I) = \rho_{\text{tun}}(\delta I + \bar{I})$ (*i.e.* the distribution shifted by mean current \bar{I}) together with the corresponding Gaussian (red curve), for $\Delta t = 1$ ns and $\bar{I} = 10$ nA, so that $\bar{N} = 65$ and $\gamma = \bar{N}^{-1/2} \simeq 0.12$. For clarity, in the inset we plot the same curves on logarithmic scale.

The simplest way to measure ρ_{tun} in the LF regime is through Eq. (2), valid in the limit of a step-like transition in the escape histogram P_0 . The latter condition requires that the width of the transition in the absence of noise Δ_{int} is much smaller than the width of the distribution $\sqrt{c_2}$. However, this is hardly possible with realistic parameters, since the above two quantities are typically of the same order. Indeed, in a typical experiment Δ_{int}/I_c is of the order of 1×10^{-3} , whereas with $\bar{I} = 10$ nA, $I_c = 1$ μ A and $\Delta t = 5$ ns one finds $\sqrt{c_2}/I_c = 6 \times 10^{-4}$ [31]. Surprisingly, the derivative of the probability histogram P_{tun} , obtained by Eq. (1) using ρ_{tun} , turns out to be distinguishable from the corresponding derivative of P_{Gauss} , relative to a Gaussian distribution of same width. This can be appreciated in the left-hand-side of the plot reported in Fig. 5, where the above two quantities are plotted as a

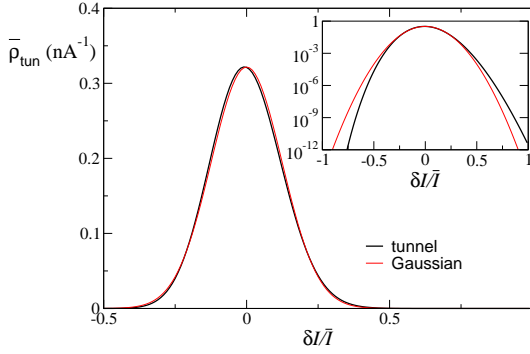


Fig. 4: Distribution of current fluctuations $\bar{\rho}_{\text{tun}}(\delta I)$ relative to a tunnel junction with $\Delta t = 1$ ns and $\bar{I} = 10$ nA (black curve) and of the corresponding Gaussian of equal width (red curve). In the inset we plot the same curves in logarithmic scale.

function of the normalized current fluctuations. Even in this far-from-ideal situation, the effect of higher cumulants is weak but present [32]. It is worthwhile mentioning that, in order to make use of Eq. (2), one needs to amplify the cumulants by a factor α , for example through an external circuit. For the figures reported above, a value of $\alpha = 100$ appears to be sufficient for a determination of ρ_{tun} . We shall see in the next paragraph that the LF regime is much more suited for peculiar distributions.

Flux random telegraph noise. — Next we consider the situation where the flux threading the dc-SQUID in Fig. 1 is affected by random telegraph noise (RTN) and, for definiteness, where the current is noiseless. The possibility of such a source of noise has been considered in Ref. [33]. The time-dependent flux can be written as $\phi(t) = \bar{\phi} + \Delta\phi \cos[\pi \sum_k \theta(t - t_k)]$, where $\bar{\phi}$ is the average flux, $\Delta\phi$ is the displacement and t_k are randomly distributed times. The distribution of flux fluctuations $\delta\phi$ is given by

$$\rho_{\text{flux}}(\delta\phi) = \frac{1}{2} [\delta(\delta\phi - \Delta\phi) + \delta(\delta\phi + \Delta\phi)], \quad (12)$$

with non-zero even central moments ($\langle (\delta\phi)^{2n} \rangle = (\Delta\phi)^{2n}$) and characteristic function $\chi(\lambda) = \frac{1}{2} (e^{i\lambda\Delta\phi} + e^{-i\lambda\Delta\phi})$. The time-dependent correlation function is given by $\langle \delta\phi(t)\delta\phi(0) \rangle = (\Delta\phi)^2 e^{-2|t|p}$, where p is the probability of transition between the two states ($+\Delta\phi/2, -\Delta\phi/2$) per unit time. The noise power (double Fourier transform) is therefore a Lorentzian $S(\omega) = \frac{(\Delta\phi)^2}{2\pi} \frac{2p}{4p^2 + \omega^2}$. In this case one can choose the values of p and Δt such that flux fluctuations has only a LF component. Indeed, if we assume that $1/\Delta t \gg p$, the escape probability is given by

$$P(I, \bar{\phi}) = \int d(\delta\phi) \rho_{\text{flux}}(\delta\phi) P_0(I, \bar{\phi} + \delta\phi), \quad (13)$$

in the case where there is no current noise. In Eq. (13), the flux dependence in $P_0(I, \phi)$ is introduced by taking into account the fact that $I_c = 2I_{\text{cJJ}} \left| \cos\left(\frac{\pi\phi}{\phi_0}\right) \right|$, where I_{cJJ} is the critical current of a single JJ. In Fig. 6, $P(I, \bar{\phi})$ is

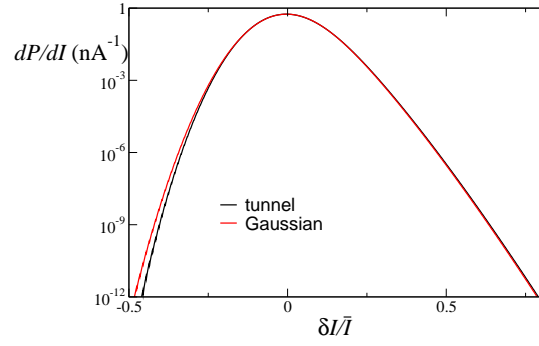


Fig. 5: Logarithmic plot of the derivative of P_{tun} relative to a tunnel junction (black curve) for realistic parameters describing the JJ (see text). In red is plotted the corresponding curve for a Gaussian current distribution of equal width, which can be distinguished from the black curve, in the range of values considered, at least for negative fluctuations.

plotted for $\bar{\phi}/\phi_0 = 1/5$, $\Delta\phi/\phi_0 = 1/60$, $I_c = 1\mu\text{A}$, $\Delta t = 5$ ns and $E_C/E_J = 10^{-4}$ ($\phi_0 = h/2e$ being the flux quantum). The red curve refers to the absence of noise, the black curve to RTN and blue curve to Gaussian noise (of equal width). The escape histogram relative to the RTN presents a double step structure, well distinguishable from the histogram relative to Gaussian noise. The double step histogram can be understood as due to averaging two histograms with different escape current, once a finite average flux $\bar{\phi}$ is assumed. However, the effect of RTN noise in the HF regime leads to a decreased escape current. The general feature of the HF regime is confirmed also in this case: the effect of higher cumulants on the histogram reduces to a shift in the escape probability, see also Ref. [11].

Conclusions. — To summarize, we have shown that the escape probability histogram of an underdamped current-biased dc-SQUID, operated with very short current pulses, can be used for retrieving information on the distribution of current fluctuations which characterizes a given conductor. The latter is connected to the dc-SQUID through some filtering circuit, which we do not consider, for the sake of definiteness, but would affect the measurement. More precisely, this method is sensitive to the low frequency component of the fluctuations, namely up to $1/\Delta t$ (Δt being the duration of the pulse, of the order of a few nanoseconds). We have first analyzed the case in which the distribution presents a small third moment (small skewness limit), while all other moments are vanishing. In particular, we have addressed the functional dependence of escape current and width of the transition with respect to the skewness, and found that, even for short pulses, the value of the skewness can be in principle evaluated by inverting the polarity of the bias current. We have then analyzed two concrete examples: in the first one we have assumed that the current fluctuations originates from a tunnel junction and we have evaluated the possibility of determining the whole current distribution, by

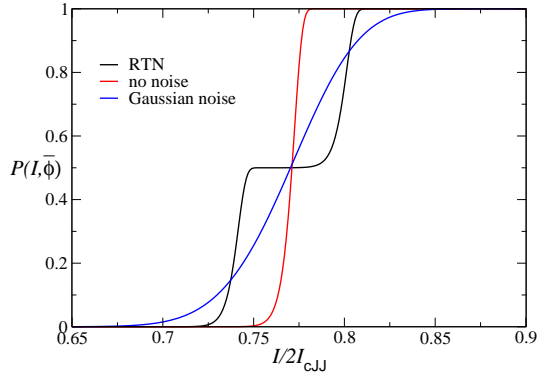


Fig. 6: Probability histograms with flux LF noise with $\bar{\phi}/\phi_0 = 1/5$, $\Delta\phi/\phi_0 = 1/60$, $I_c = 1\mu\text{A}$, $\Delta t = 5\text{ ns}$ and $E_C/E_J = 10^{-4}$. Black curve is relative to RTN, blue curve to Gaussian noise and red curve to no noise. I is expressed in units of $2I_{cJJ}$.

simulating the probability histogram under realistic conditions. We wish to mention that the same analysis can be performed for any mesoscopic element. In the second example, we have considered flux fluctuations characterized by a random telegraph noise distribution, finding that peculiar distributions such as the latter produce a very clear signature, not present if the measurement is performed with long current pulses. Finally we wish to stress that we focused the analysis to current pulses only, even though it remains valid for the case of flux pulses.

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- [25] For instance induced by a fluctuating magnetic flux threading the weak link.
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- [31] We assume $E_C/E_J \simeq 1 \times 10^{-7}$, which yields a value of $\Delta I_{\text{int}}/I_c$ compatible with the experimental one.
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